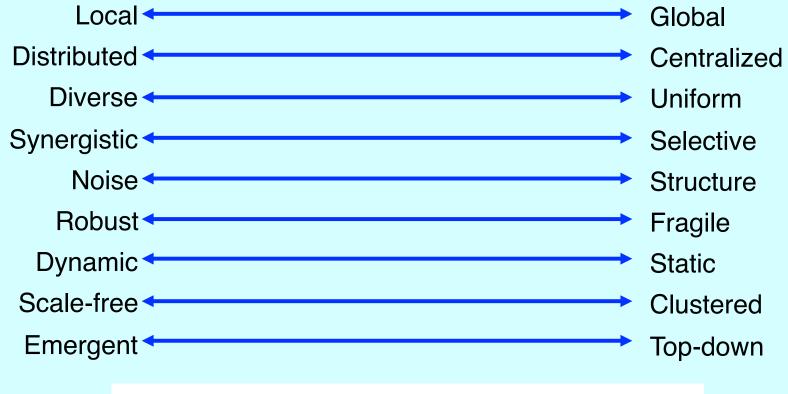


"Role of Scaling in Developing an Understanding of How Systems Work"

### Dr. Norman L. Johnson

norman@santafe.edu http://CollectiveScience.com

# How "SFI-ish" is your System?



#### Qualities to consider in the above measures

- Dimensionality (spatially, functionally)
- Connectivity Access to information
- Governing equations / Rules
- Fitness function, quality/performance measures
- Scalability

# **Power Laws and Complexity:** Not without some controversy

"Over the last decade or so, it has become reasonably common to see people (**especially physicists**) claiming that some system or other is complex, because it exhibits a power law distribution of event sizes. Despite its popularity, this is simply a fallacy. No one has demonstrated any relation between power laws and any kind of formal complexity measure. Nor is there any link tying power laws to our intuitive idea of complex systems as ones with strongly interdependent parts."

In "METHODS AND TECHNIQUES OFCOMPLEX SYSTEMS SCIENCE: AN OVERVIEW" by Cosma Shalizi

# **My Background**

**Polymer physics** 

**Star Wars** 

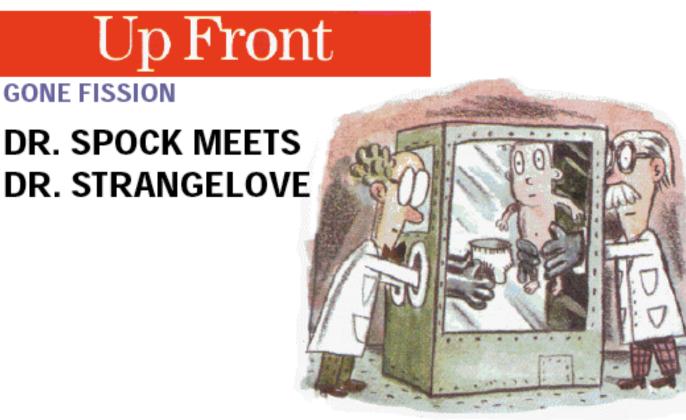
Novel fusion device

**Combustion modeling** 

Hydrogen Fuel Program

**P&G multi-phase flows** 

# **P&G Application Area?**



BUSINESS WEEK / July 10, 2000

# **My Background**

**Polymer physics** 

**Star Wars** 

**Novel fusion device** 

**Combustion modeling** 

Hydrogen Fuel Program

**P&G multi-phase flows** 

Biological threat reduction & homeland security

**Bird Flu - Mitigation development** 

# **My Real Background**

**Polymer physics** 

**Star Wars** 

**Novel fusion device** 

**Combustion modeling** 

Hydrogen Fuel Program

**P&G multi-phase flows** 

Biological threat reduction & homeland security

Bird Flu - Mitigation development

Future of the internet

Self-organizing collectives

Diversity and emergent problem solving

• Finance applications

#### **Developmental view of evolution**

• Finance applications

#### Failure of Experts

Effects of rapid change on collectives

• Finance applications

Identity formation and interaction

- Psycho-social simulation models
- Coexistence applications

# **Challenges Facing Business**

Increased 'complexity' (too vague) Change happening faster and faster **Data-Poor to Data-Rich environment** "Fall of the House of Experts" **Globalization - connected markets Technology surprise / reset** ??

# **Opportunities Facing Business**

Increased 'complexity'

Faster Change

**Data-Rich environment** 

"Fall of the House of Experts"

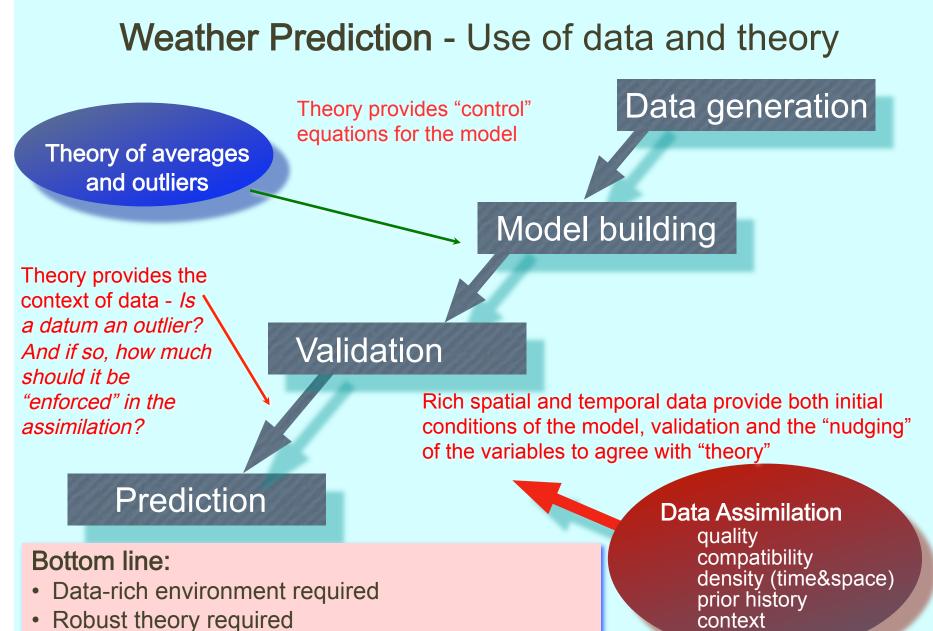
**Globalization - connected markets** 

**Technology surprise** 

Success for adaptive businesses "Creative destruction" Data-driven solutions Collective solutions arkets Expanding markets Large payoffs

Above opportunities are not just additive

All required understanding changing data, context, & knowledge Survival Guide 2006



- Ideal for chaotic or high-uncertainty systems
- Rich interaction between data, analysis and theory

### Origin of "The Theory"

### Data generation

Theory of averages and outliers

### Discovery

### Increasing levels of discovery:

- Statistical characterization
- Dimensionless functionality (correlations)
- Scaling self-similarity
- Descriptive "Laws"
- Functional relationships
  - Static
  - Dynamic (governing equations of change)
- Higher moments (variation within)
- Error generation

# Drucker's Age of Discontinuity (1968)

### Method: Observe trends and predict changes, not details

- New Global information economy Knowledge is new capital
  - New technologies create new technologies
  - High mobility of men, capital, and ideas
- New pluralism (diversity)
  - Will make obsolete old theories of economy, government and society
- New structure based on social responsibility and accountability

Drucker's prediction of globalization and faster change was 28 years ago, in the time of greatest IT change.

### The context in 1968

"In fact, IBM currently is selling 100s of computers a month."

# Power Laws and Complexity (con't)

"... it has been known for half a century that there are many, many ways of generating power laws, just as there are many mechanisms which can produce Poisson distributions, or Gaussians.

"Perhaps the simplest one is that recently demonstrated by Reed and Hughes, namely exponential growth coupled with random observation times.

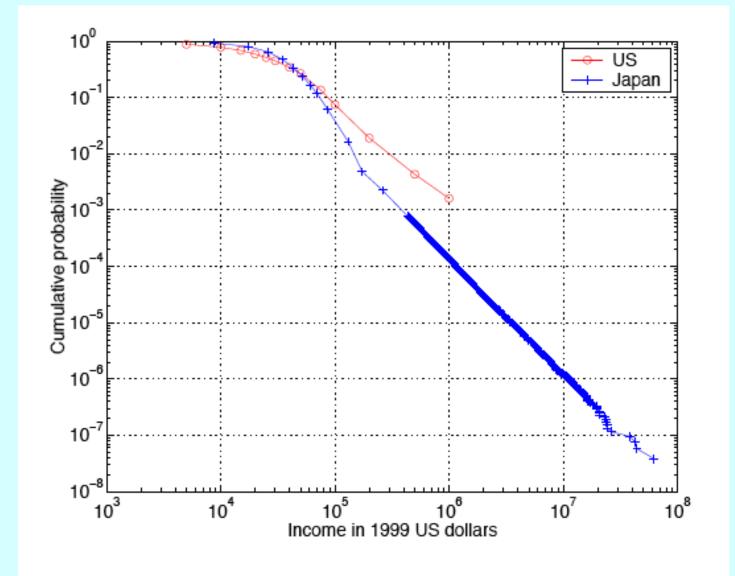
"The observation of power laws alone thus says nothing about complexity (except in thermodynamic equilibrium), and certainly is not a reliable sign of some specific favored mechanism, such as self-organized criticality or highly-optimized tolerance."

In "METHODS AND TECHNIQUES OFCOMPLEX SYSTEMS SCIENCE: AN OVERVIEW" by Cosma Shalizi, Center for the Study of Complex Systems, University of Michigan, Ann Arbor

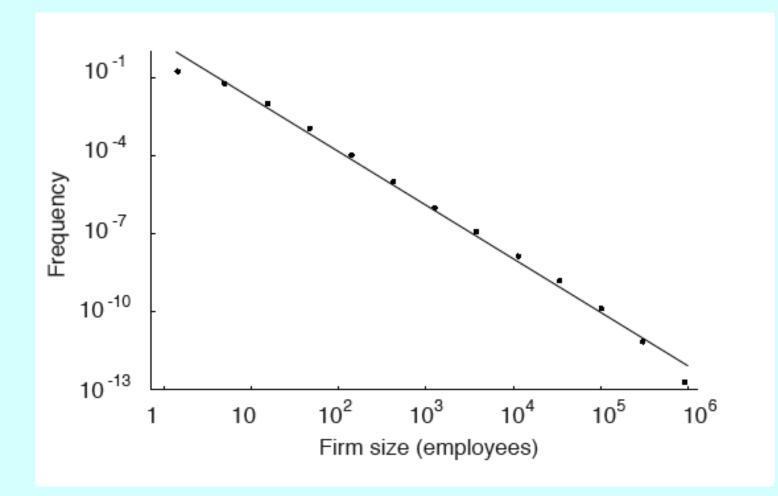
# **Importance of Power Laws - Farmer**

"Scale free behavior has important scientific implications because it strongly suggests that the same mechanism is at work across a range of different scales."

# Income distribution in US and Japan



# US Firm Size by Employees

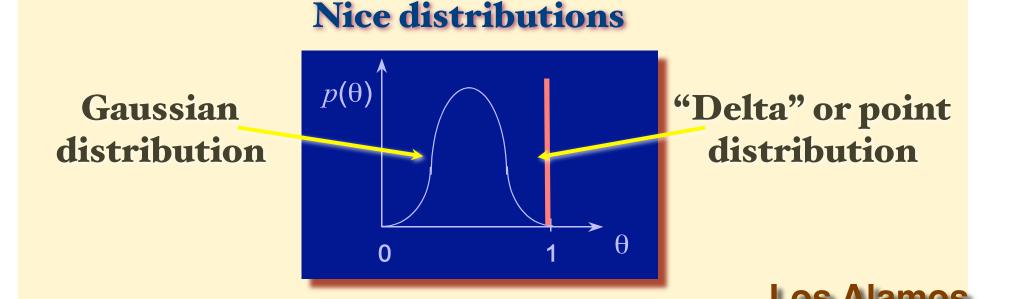


# Why care about distributions?

One approach to prediction is a dynamical theory, e.g., non-equilibrium statistical mechanics

**Analytical forms of dynamical theories require "nice" distributions for probability distribution functions** (PDFs)

PDFs map variables into observables



Normal (Gaussian) distribution

$$P(x) \sim \exp(-\frac{x^2}{2\sigma^2}).$$

Defined on  $(-\infty, \infty)$ 

**Natural explanation**: Central limit theorem: Sum of many random variables (second moment finite).

**Many applications**: Maxwell: velocity distribution of particles in a gas Heights of individuals, IQ, ...

Distribution is thin tailed: No one is 10 feet tall

Normal (Gaussian) distribution

$$P(x) \sim \exp(-\frac{x^2}{2\sigma^2}).$$

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**Natural explanation**: Central limit theorem: Sum of many random variables (second moment finite).

**Many applications**: Maxwell: velocity distribution of particles in a gas Heights of individuals, IQ, ...

Distribution is thin tailed: No one is 10 feet tall

**Exponential distribution** 

$$P(x) \sim \exp(-x/x_0)$$

Defined on  $[0, \infty)$ 

Natural explanation (1): Survival times for constant probability of decay

Natural explanation (2): Equilibrium statistical mechanics (Maximum entropy subject to constraint on mean)

Many applications: Radioactive decay, Energy distribution at equilibrium, ...

**Distribution is also thin tailed:** characteristic scale  $x_0$ .

Power law

$$P(x) \sim x^{-\alpha}$$

Defined on [a,  $\infty$ ).  $\alpha$ , a > 0.

Natural explanation?

Distribution is heavy tailed, no characteristic scale.

Many applications – is there a common link?

A power law is a linear relation between logarithms

$$f(x) = Kx^{-\alpha}$$
$$\log f(x) = -\alpha \log x + \log K$$

### Difference between Thin and Thick Tailed distributions

Similar for frequent events

1/1000 event is twice as large for a power law

1/10,000 event is three and a half times as large

"The probability of observing a fluctuation of 21% (the size of the famous negative S&P return on October 19, 1987) under the normal hypothesis is less than 10<sup>-16</sup>, whereas the probability under the power law distribution is 0.08%. Under the normal distribution it is essentially impossible that this event could ever have occurred, whereas under a power law distribution such an event is to be expected." - Farmer

### **Many Examples of Power-Law Distributions**

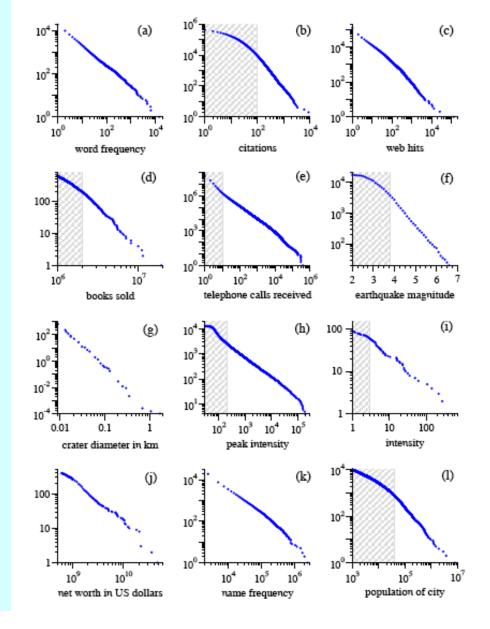
#### Frequency vs. measure

earthquakes flood levels rainfall craters insurance claims income price changes firm size transaction volume city size intensity of wars length of strikes

DoD cost overruns frequency of word usage name frequency authored papers citations to papers patent profitability music sales movie sales book sales telephone calls price for order placement number of email messages size of computer files hits on web page links to web sites

### **Many Examples of Power-Law Distributions**

Frequency vs. measure



# What's Special about Power Laws

### **Power laws are scale invariant**

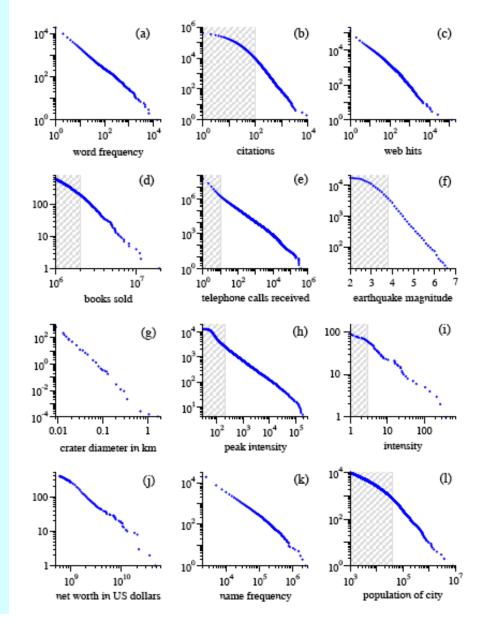
• Retains functional form under scaling

### Power law is an asymptotic relation

- Only meaning full in a bounded region
- Practically, real problems always have cut-offs

### **Unshaded region - Asymptotic**

Frequency vs. measure



# What's Special about Power Laws

#### Power laws are scale invariant

• Retains functional form under scaling

#### Power law is an asymptotic relation

- Only meaning full in a bounded region
- Practically, real problems always have cut-offs

#### Power laws have a threshold above which moments don't exist

• For this reason there is no such thing as an "average flood" - it is only possible to measure flood likelihoods in terms of quantiles, as in the statement "this is a 100 year flood". (Farmer)

### Combinations (aggregations) of power laws remain a power law

Any "reasonable" function with moments that don't exist, i.e. "truly heavy tails", is a power law

# Power Laws vs. Long-Memory

Difficult to determine whether power laws or long-memory are present by empirical sampling

Tails by definition only have a small number of events. May not have enough data to probe tails.

Can have slow convergence to tail (slowly varying functions)

Can have cutoffs of tail due to finite size effects (e.g. physical limits)

Thin tailed distributions may mimic power law behavior over a wide range

Statistical convergence for long-memory processes is very slow

# Importance of Power Laws - Farmer

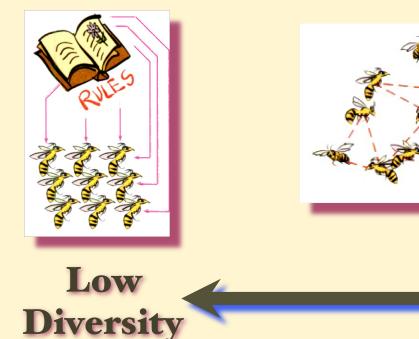
- "Scale free behavior has important scientific implications because it strongly suggests that the same mechanism is at work across a range of different scales."
- "The real test is whether power laws can improve our predictive or explanatory power by leading to better models. Self-similarity is such a strong constraint that, even if only an approximation over a finite range, it is an important clue about mechanism."
- "Ultimately, the best method to demonstrate that power laws are applicable is to construct theories that also have more detailed testable predictions."

**Diversity and Collective Prediction** Prediction of *collective* behavior is generally easier at extremes of diversity or variation

Locally and Globally Predictable

Unpredictable

Globally Predictable

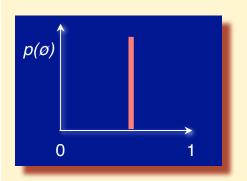






**Diversity and Collective Prediction** How does this translate to distribution functions?

### Locally and Globally Predictable



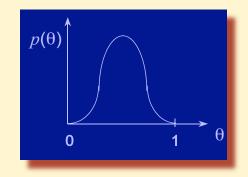
### Unpredictable

### **Problem distributions:**

- Discrete distributions
- Multi-modal distributions
- Long-tailed distributions
  - (e.g., power law,

instead of Gaussian statistics)

### **Globally Predictable**





# **Mechanisms for Generating Power Laws**

Critical points and deterministic dynamics

Random processes

Sampling from (terminated) exponential growth (Reed and Hughes)

**Mixtures** 

**Dimensional constraints** 

Maximization principles

Preferential attachment

Non-equilibrium statistical mechanics

# **Causes of Anomalous Distributions**

**Incomplete sampling** Habitual behavior **Non-equilibrium dynamics** Co-evolving systems - markets "Structures" limit options Regulations, cultures, ... **Boundary effects dominate** Behavior at borders **Coupling or interdependence of different levels** Hierarchical organizations **Clustering or localized regions of interaction** Typical of social, information and power networks

# Statistical Issues Relating to Power Laws "some common mistakes"

#### **Parameter Estimation**

Use linear regression to find the line of best fit to the points on the log-log plot. But the line minimizing the sum of squared errors is not a valid probability distribution, and so this is simply not a reliable way to estimate the distribution.

#### **Error Estimation**

Estimate of the standard error in the estimated slope and report this as the uncertainty in the power law. "This is an entirely unacceptable procedure." On a log-log plot this violates the assumption that measured values have Gaussian fluctuations around their true means.

#### Validation

"The basic problem here is that any smooth curve looks like a straight line, if you confine your attention to a sufficiently small region and for some nonpower-law distributions, such sufficiently small tregions can extend over multiple orders of magnitude.

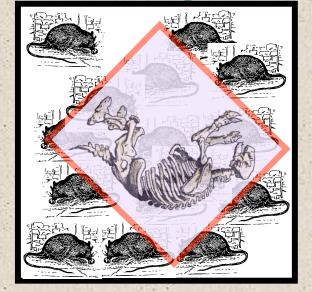
In "METHODS AND TECHNIQUES OFCOMPLEX SYSTEMS SCIENCE: AN OVERVIEW" by Cosma Shalizi

# Rat Studies of Maximum Carrying Capacity

**Cooperative social structure** 



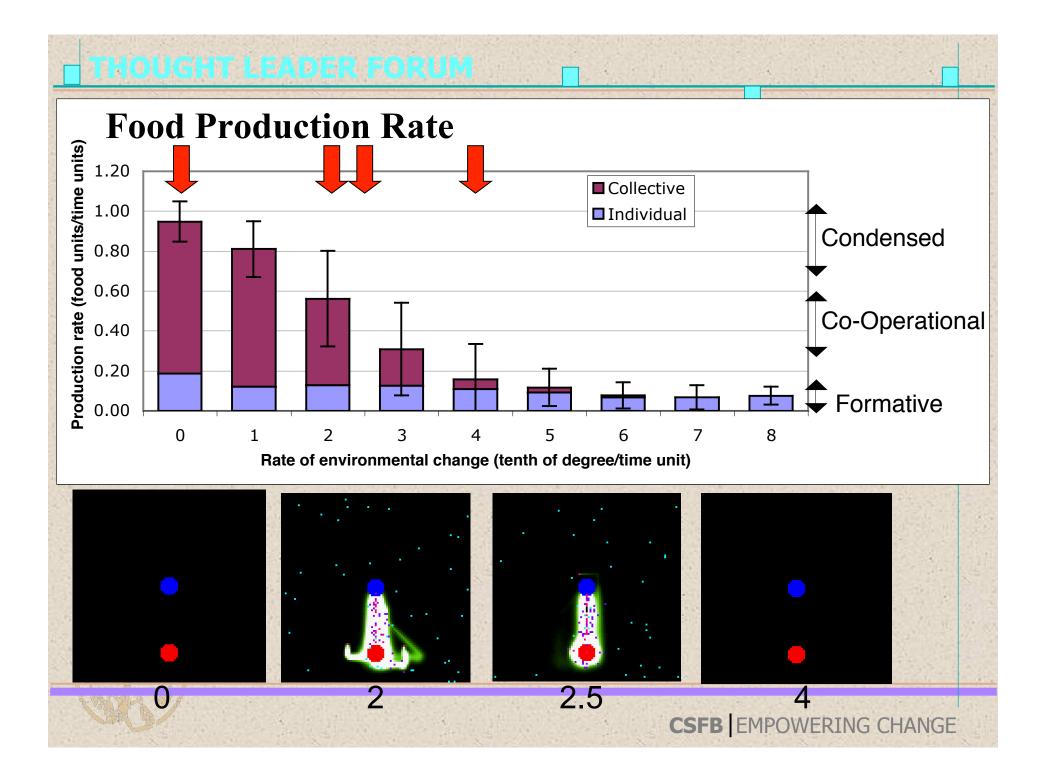
**Control - no imposed social structure** 

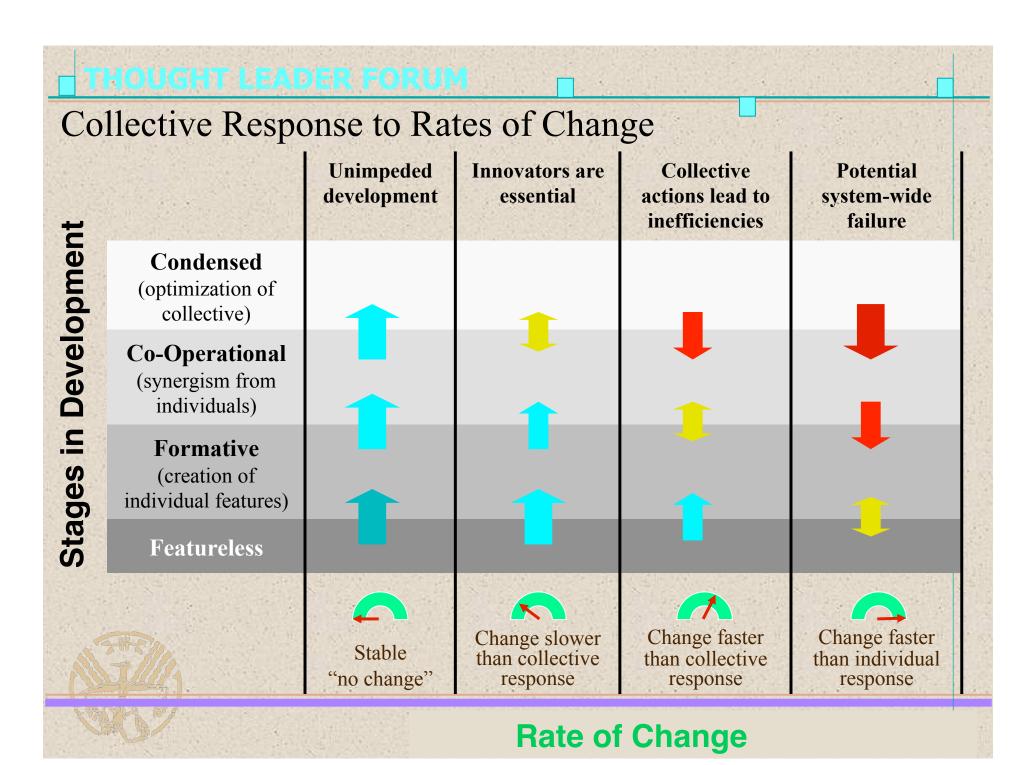


NIMH psychologist John B. Calhoun, 1971

Both systems loaded to 2 1/2 times the optimal capacity. Social order system can carry 8 times the optimal capacity.

CSFB EMPOWERING CHANGE



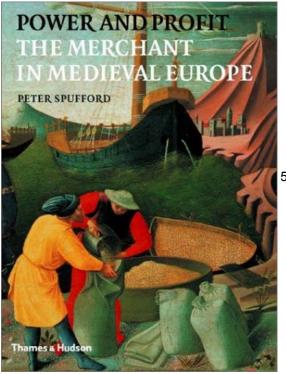


## On Constructing Theories (Models) and Testing

- 1. Replication is essential.
- 2. It is a good idea to share not just data but programs.
- 3. Always test the robustness of your model to changes in its parameters. (This is fairly common.)
- 4. Always test your model for robustness to small changes in qualitative assumptions. If your model calls for a given effect, there are usually several mechanisms which could accomplish it. If it does not matter which mechanism you actually use, the result is that much more robust. Conversely, if it does matter, the over-all adequacy of the model can be tested by checking whether that mechanism is actually present in the system. Altogether too few people perform such tests.
- In "METHODS AND TECHNIQUES OFCOMPLEX SYSTEMS SCIENCE: AN OVERVIEW" by Cosma Shalizi

European Conference on Complex Systems Paris, 14-18 November 2005

# Civilizations as dynamic networks



Cities, hinterlands, populations, industries, trade and conflict

Douglas R. White © 2005 All rights reserved

50 slides - also viewable on drw conference paper website version 1.3 of 11/12/2005

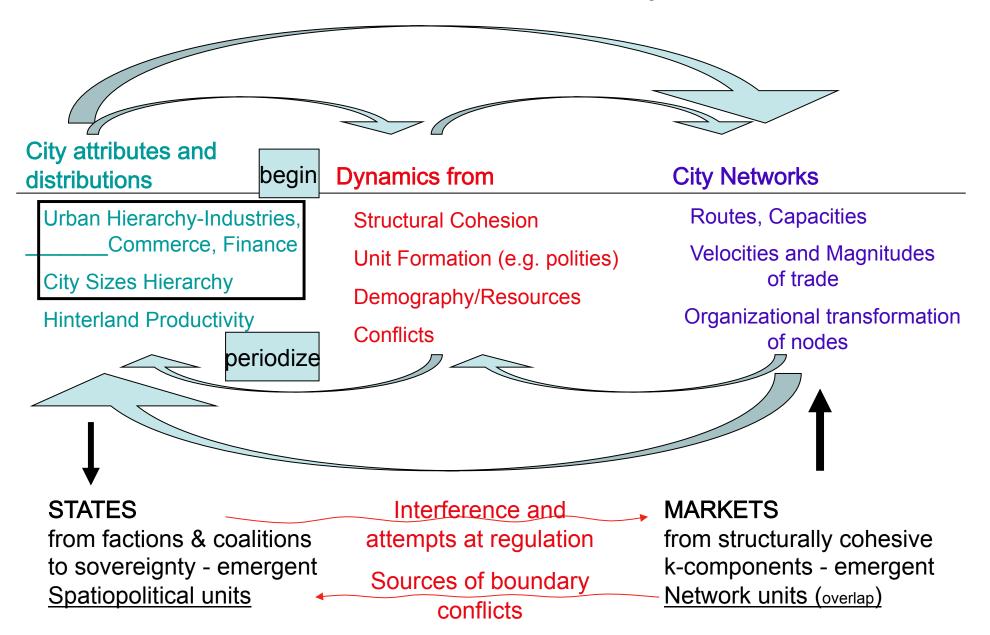


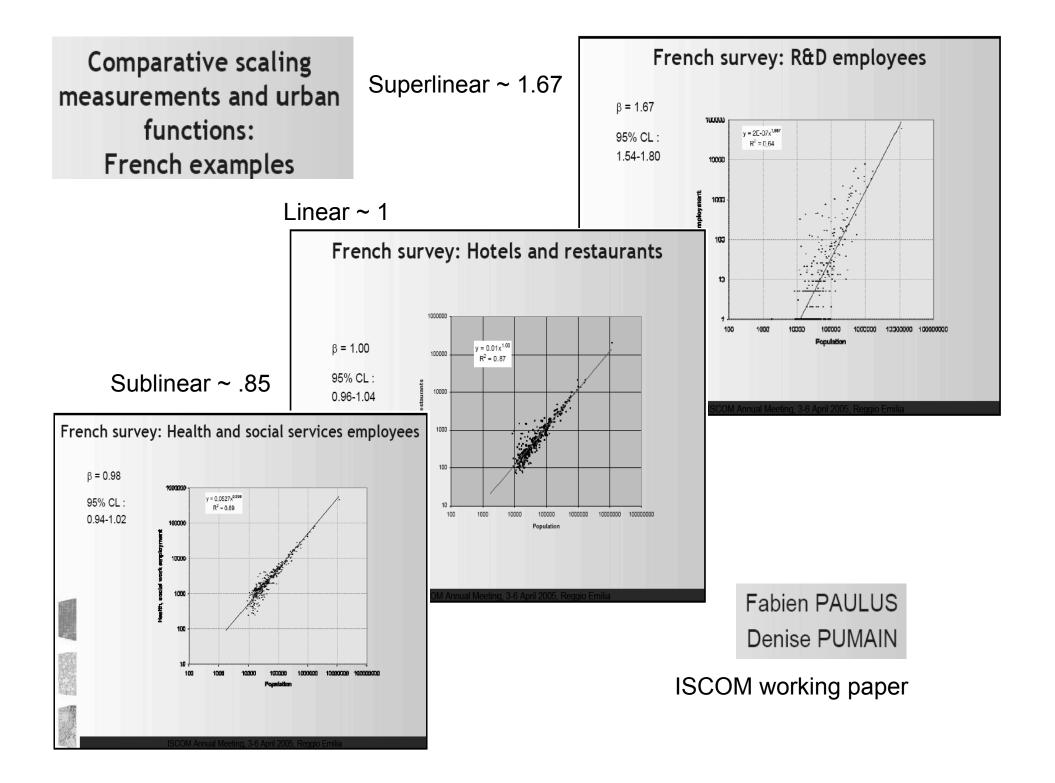
Medieval to Modern: Civilizations as Dynamic Networks

> Douglas R. White and Peter Spufford © Douglas R. White, All rights reserved



### **Co-evolution time-series of Cities and City Networks**

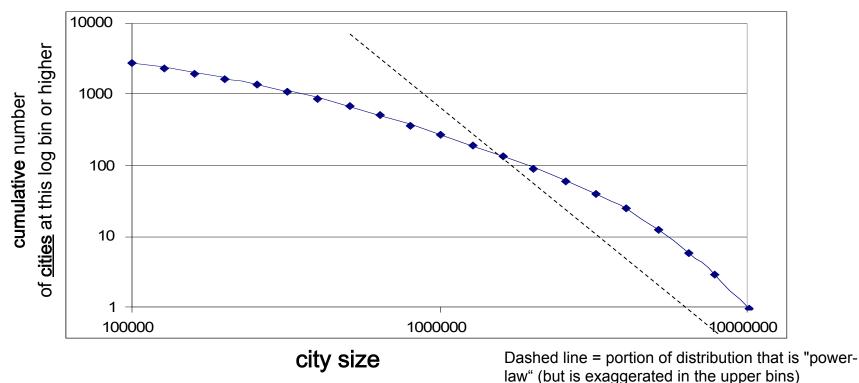




#### Urban Scaling: City Sizes - 1950 United Nations data for world cities

Compare the scale K and  $\alpha$  coefficients of the power-law  $y(x) \approx K x^{-\alpha}$  (and Pareto  $\beta = \alpha + 1$ ) with the *q*-exponential parameters for *q* slope and scale  $\kappa$  in  $y(x) \sim [1 + (1-q) x/\kappa)]^{1/(1-q)}$ , fitted to entire size curves

Power laws and Zipf's law might fit upper bin frequencies for city sizes but not the whole curve



(White, Kejžar, Tsallis, and Rozenblat © 2005 working paper)

### Origin of "The Theory"

### Data generation

Theory of averages and outliers

### Discovery

#### Increasing levels of discovery:

- Statistical characterization
- Dimensionless functionality (correlations)
- Scaling self-similarity
- Descriptive "Laws"
- Functional relationships
  - Static
  - Dynamic (governing equations of change)
- Higher moments (variation within)
- Error generation

### Summary

Why true power law observations are exciting

Essential to discovery of governing "laws" Often from a critical process in a complex system

But power laws are only part of the process to developing a broader understanding. Not meaningful without a model or theory.

Power laws behavior doesn't mean complexity

**Power laws are** 

(Idealized) asymptotic behavior - beware of the boundaries! Represent thick tails, rather than thin tailed distributions

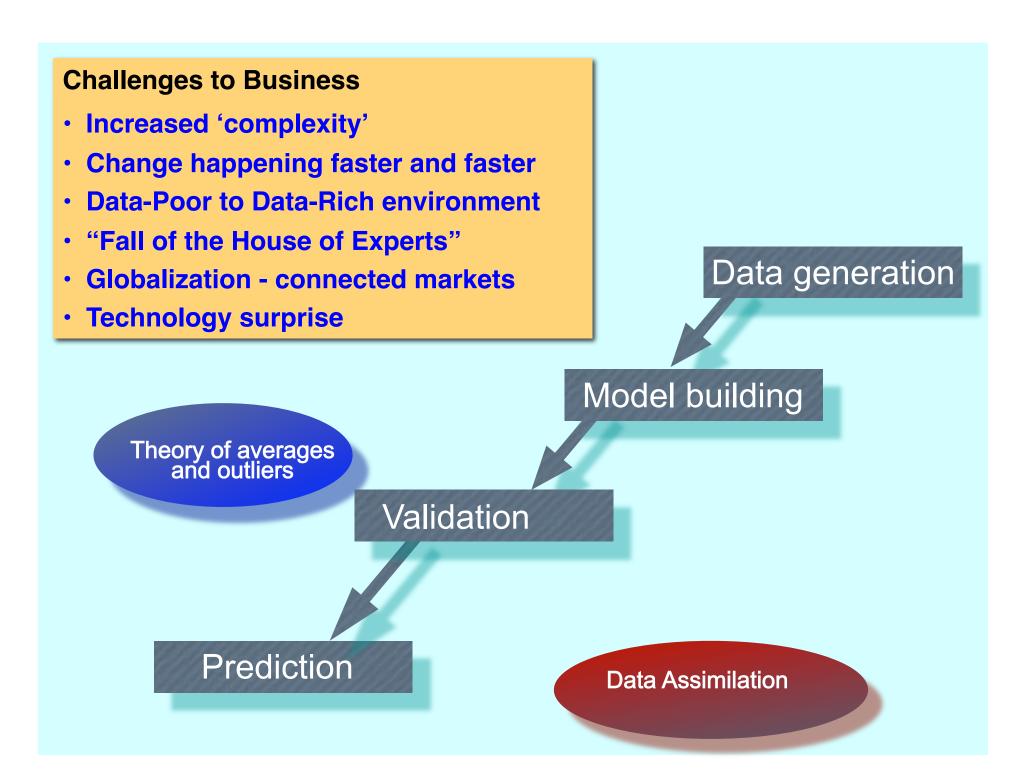
In general, anomalous distributions may dominate real systems

Consider maturity of system evolution - "young" systems may be very different

Or mixed maturity of sub-systems

**Consider optimization versus robustness** 

Best resources on the subject (appear to be) from SFI



### References

Shalizi, Cosma R., "METHODS AND TECHNIQUES OF COMPLEX SYSTEMS SCIENCE: AN OVERVIEW", Chapter 1 (pp. 33-114) in Thomas S. Deisboeck and J. Yasha Kresh (eds.), Complex Systems Science in Biomedicine (New York:Springer, 2006) http://arxiv.org/abs/nlin/0307015

- Farmer, J. Doyne & John Geanakoplos, "Power laws in economics and elsewhere", DRAFT April 4, 2005 (chapter from a preliminary draft of a book called "Beyond equilibrium and efficiency") Contact the authors for a copy.
- Farmer, J. Doyne, "Power laws", Santa Fe Institute Summer School June 29, 2005. Contact the author for a copy.
- Holbrook, Morris B. 2003. "Adventures in Complexity: An Essay on Dynamic Open Complex Adaptive Systems, Butterfly Effects, Self-Organizing Order, Co-evolution, the Ecological Perspective, Fitness Landscapes, Market Spaces, Emergent Beauty at the Edge of Chaos, and All That Jazz http://www.amsreview.org/articles/holbrook06-2003.pdf
- White, Douglas R., "Civilizations as dynamic networks: Cities, hinterlands, populations, industries, trade and conflict", <u>European Conference on Complex Systems</u> Paris, 14-18 November 2005. http://eclectic.ss.uci.edu/~drwhite/ ppt/CivilizationsasDynamicNetworksParis.ppt

For exceptional talks on Complexity in financial systems, see the Thought Leaders Forums:

- http://www.leggmason.com/thoughtleaderforum/2006/index.asp for 2003-2006
- http://www.capatcolumbia.com/CSFB%20Thought%20Leader%20Forum.htm for 2000-2003

### **Topics for Discussion?**

What are the implications in your application area of the shift to being "data rich"? Are the moments around the norm important? Are you driven by the outliers or norms? How much can the results presented today be generalized?

Stephanie's table of correspondence? Worth more development / study? Are global properties always determined by the "supply chain" - as Geoff observed in organisms? How do physical/operational restrictions dominate the growth of a system? In complex composite systems, does the host always dominate the dynamics?

Can human-designed systems be better planned by using general "laws"? Bio-inspired solutions - where/when valid?

Are ant colonies more like bio organisms or human societies? What about material versus information differences?

(If I give you an apple, I don't have an apple; is information the same, why not?) How does basic scales affect the system: e.g., cell size is fixed, but computer size is not What can be concluded about network scaling and global system performance?

#### Social systems and innovative reset (Singularity horizon?)

Assumes infrastructure is largely unchanged

Can innovation reset infrastructure?

#### What about optimization of Efficiency versus Robustness?

In times of faster change, robustness is maybe more important

#### Stages of development and maturity

How much what has been observed today is only for mature, well developed systems? How do early innovative systems behave? How do opportunities and approaches differ? **Others?**